PHYS 798C Spring 2024
Lecture 15 Summary

Prof. Steven Anlage

I. GINZBURG LANDAU THEORY OF SUPERCONDUCTORS PART II

The Ginzburg-Landau (GL) theory of superconductivity is one of the most useful tools for doing quantitative calculations.

A. Boundary Conditions on the GL $\psi(r)$

What boundary conditions (BC) apply in an inhomogeneous superconductor? Enforce the condition that no super-current flows through the interface with a non-superconducting material (e.g. vacuum), $\widehat{n} \cdot \vec{J}_s = 0$, where $\widehat{n}$ is unit vector in the outward normal direction from the superconductor. With $\vec{J}_s = e^*|\psi|^2\vec{v}_s$ one could naively let $\psi(r)$ go to zero at the interface to satisfy the condition. However this would imply that $\hat{n} \times \vec{J}_s = 0$, meaning that the screening current parallel to the interface is also zero, which is not reasonable for a superconductor/vacuum interface, for example.

P. G. de Gennes came up with a more useful and flexible boundary condition. Using the expression for the current density $\vec{J}_s = \frac{e^*}{m^*} Re\left\{\psi^*\left(\frac{\hbar}{i}\vec{\nabla} - e^*\vec{A}\right)\psi\right\}$, one can note that:
$\widehat{n} \cdot \vec{J}_s = \frac{e^*}{m^*} Re\left\{\psi^*\widehat{n} \cdot \left(\frac{\hbar}{i}\vec{\nabla} - e^*\vec{A}\right)\psi\right\}$. Now if we make
$\widehat{n} \cdot \left(\frac{\hbar}{i}\vec{\nabla} - e^*\vec{A}\right)\psi = \frac{i}{b}\psi$, with b real and positive, then we automatically satisfy the boundary condition.

How to interpret this constraint? In the absence of a vector potential and in one dimension, it boils down to
$\frac{\partial\psi}{\partial x}|_{x=0} = -\frac{1}{b}\psi|_{x=0}$, where the boundary is assumed to be at $x = 0$. The length scale b is called the **extrapolation length**.
In the case of an insulator, $b = \infty$, and there is no suppression of the order parameter at the S/I interface.
In the case of a normal metal b is finite and the order parameter is suppressed at the S/N interface, linearly extrapolating to zero at a distance b in to the normal metal.
For a ferromagnet, one can take $b = 0$ so that the order parameter is suppressed to zero at the SC/FM interface.

B. GL Coherence Length

Starting from the GL equation with $\vec{A} = 0$, one can divide through by α and ψ_∞ to obtain
$f - f^3 - \frac{\hbar^2}{2m^*\alpha}\nabla^2 f = 0$, where $f = \psi/\psi_\infty$ is the dimensionless order parameter (and not to be confused with the free energy density!).
The pre-factor on the Laplacian must have the dimensions of length squared, and so we define the Ginzburg-Landau coherence length as $\xi_{GL}^2 \equiv \frac{\hbar^2}{2m^*|\alpha|}$. As temperature approaches T_c, α goes to zero and the GL coherence length diverges.

Solutions to the GL equation: $f - f^3 - \xi_{GL}^2 \nabla^2 f = 0$ for small perturbations from $f = 1$ yield one-dimensional solutions of the form $f = 1 - e^{\pm\sqrt{2}x/\xi_{GL}}$, showing that ξ_{GL} is the "healing length" of the order parameter.

We then compared ξ_{GL} to the (temperature independent) BCS coherence length $\xi_0 = \frac{\hbar v_F}{\pi\Delta(0)}$. The definition of the BCS coherence length arises from a treatment of the nonlocal electrodynamics of superconductors, very similar to Pippard's treatment discussed in Lecture 3. The two coherence lengths are related as
$\frac{\xi_{GL}(T)}{\xi_0} = \frac{\pi}{2\sqrt{3}}\frac{\lambda_L H_c(0)}{\lambda_{eff}(T)H_c(T)}$.
This shows that the two length scales are roughly comparable at zero temperature, at least in the clean

local limit.
Some typical values for the BCS coherence length (based on values of v_F and $\Delta(0)$) are $\xi_0 = 1.25\ \mu m$ for Aluminum, $\xi_0 = 94\ nm$ for Niobium, and $\xi_0 = 2.6\ nm$ for the cuprate superconductor YBCO.

C. Electrodynamics in the Clean and Dirty Local Limits

We considered the Drude model for local electrodynamics in which $J = \sigma E$ and $\sigma = \sigma_1 - i\sigma_2$. σ_1 measures the dissipative part of the complex conductivity. That is, it measures the current that flows in phase with the electric field. σ_2 measures the reactive part of the complex conductivity. That is, it measures the current that flows in quadrature with the electric field.
We found that in the clean limit $\ell_{MFP} >> \xi_0$ that $\lambda_{eff}(0) \approx \lambda_L$ because almost all of the oscillator strength in σ_1 condenses in to the delta function at zero frequency.
In the dirty limit $\ell_{MFP} << \xi_0$ only a fraction of the oscillator strength condenses into the delta function at zero frequency, and one finds $\lambda_{eff}(0) \approx \lambda_L \sqrt{\frac{\xi_0}{\ell_{MFP}}}$, and the effective screening length can be quite long. This is the case for superconductors like amorphous Mo-Ge, granular Al, and NbN.

D. Ginzburg-Landau κ Parameter

The dimensionless GL κ parameter is defined as

$$\kappa \equiv \frac{\lambda_{eff}(T)}{\xi_{GL}(T)}.$$

It is found that κ is nearly temperature independent near T_c.
Metals like Al have $\kappa << 1$ and are called type-I superconductors.
Cuprates and other "high-T_c" superconductors have $\kappa >> 1$ and are called type-II superconductors. We shall see why these distinctions are made in the next lecture.

Some typical values of κ are as follows.
Type I ($\kappa < 1/\sqrt{2}$)
For Al, $\lambda_L \approx 16\ nm$, $\xi_0 \approx 1.25\ \mu m$, giving $\kappa = 0.01$.
For Sn, $\lambda_L \approx 35\ nm$, $\xi_0 \approx 300\ nm$, giving $\kappa = 0.11$.
Type II ($\kappa > 1/\sqrt{2}$)
For YBCO, $\lambda_L \approx 150\ nm$, $\xi_0 \approx 2.6\ nm$, giving $\kappa = 58$.